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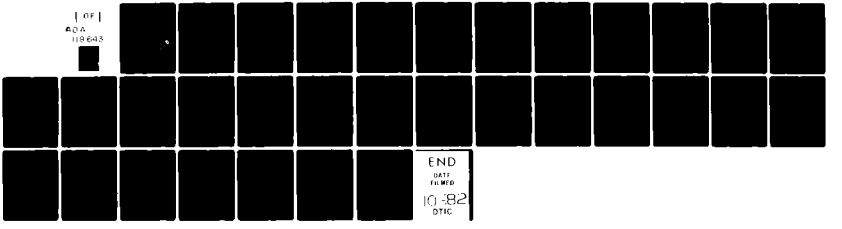

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A computational algorithm utilizing the free vibration modes of a structure is presented for calculating receptances. The usual eigensystem computed for large structural models is incomplete; hence the receptances are approximate. The formulas developed here increased receptance accuracy compared to classical spectral representations. Receptances are used extensively in eigensolution reanalysis, design and synthesis and also for forced harmonic response studies. In these areas receptance approaches offer a popular alternative to Rayleigh Ritz subspace type methods. The receptance matrix derived is applicable to

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A RECEPTANCE FORMULA FOR GENERAL SECOND DEGREE SQUARE LAMBDA MATRICES

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SUMMARY

A computational algorithm utilizing the free vibration modes of a structure is presented for calculating receptances. The usual eigensystem computed for large structural models is incomplete; hence the receptances are approximate. The formulas developed here increased receptance accuracy compared to classical spectral representations. Receptances are used extensively in eigensolution reanalysis. design and synthesis and also for forced harmonic response studies. In these areas receptance approaches offer a popular alternative to Rayleigh Ritz subspace type methods. Structural models represented by non-symmetric mass, damping and stiffness matrices, which occur frequently in rotating structures, may be treated using the receptance formulas presented. The receptance matrix derived is applicable to general, 2nd degree, square lambda matrices. This generalizes the receptance matrix commonly associated with matrix pencils.

INTRODUCTION

In the terminology of vibration theory the receptance, or dynamic flexibility influence coefficient, between degrees of freedom i and j is defined as

$G_{ij}(\omega)$ = complex amplitude of the steady state displacement at dof i due to a unit sinusoidal force of frequency ω applied at dof j.

$$= \{(-\omega^2 \underline{M} + i\omega \underline{C} + \underline{K})^{-1}\}_{ij} \qquad (1)$$

Receptances have been extensively used in dynamic reanalysis, that is in the efficient solution of the matrix eigenvalue problem for a structure undergoing physical modifications. Receptances are also used in optimized design and structural damage studies as well as in the synthesis of structures possesing prescribed eigenvalues. In the latter case this procedure is referred to as eigensolution design or "pole placement." Modal synthesis, i.e., the computation of eigensolutions of a segmented system from the eigensolutions of its component substructures, has also been presented from a receptance standpoint. The receptance methods can in some cases provide much better computational efficiency and accuracy than a standard Rayleigh Ritz subspace approach. In reanalysis this is particularly evident if the subspace dimension of the Rayleigh Ritz method is much larger than the number of degrees of freedom directly affected by the physical modification.

Bishop et. al. systematically utilized receptances in vibration theory and experimentation.[1,2,3] To calculate individual elements of a receptance matrix for an undamped system, these authors utilized its spectral representation

$$G_{ij}(\omega) = \{(-\omega^2\underline{M} + \underline{K})^{-1}\}_{ij}$$

$$= \sum_{\ell=1}^{N} \frac{\phi_{i\ell}\ \phi_{j\ell}}{\bar{m}_\ell(\Omega_\ell^2-\omega^2)} \qquad (2)$$

This relationship permits any element of the receptance matrix to be computed at various forcing frequencies (ω), without repeatedly inverting large matrices. Since the modal characteristics calculated for complex structural models are usually incomplete, the summation in equation (2) extends only over the L lowest modes. Typically,

$$0.1 < L/N < 0.3 \qquad (3)$$

Hence the spectral representation of the receptance matrix is approximate.

Receptance investigations have sought to improve the representation of equation (2) for L/N values in the range of equation (3). Hirai and Yoshimura[4] developed an alternative receptance formula useful in eigensolution reanalysis of locally modified, structural systems. Their "hybrid" representation is

$$\underline{G}(\omega)=\{\underline{I}_N+\omega^2\underline{K}^{-1}\underline{M}+\omega^4(\underline{K}^{-1}\underline{M})^2+\ldots+(\omega^2)^{R-1}(\underline{K}^{-1}\underline{M})^{R-1}\}\underline{K}^{-1}$$

$$+ \omega^{2R} \sum_{j=1}^{N} \frac{\underline{\Phi}_j\ \underline{\Phi}_j^T}{\bar{m}_j\Omega_j^{2R}(\Omega_j^2-\omega^2)} \qquad (4)$$

for R = 1,2,...

They demonstrated that the accuracy of $\underline{G}$ (ω) improves as R increases, for a fixed value of L. Leung[5] utilizes receptances in forced harmonic response analyses and eigensolution reanalysis of structural systems subjected to modified boundary conditions. His improvement to the receptance-spectral representation of equation (1) resembles the R = 2

form of equation (4).

Imbert[6] separates equation (2) into two series

$$G_{ij}(\omega) = \sum_{\ell=1}^{L} \frac{\phi_{i\ell}\,\phi_{j\ell}}{\bar{m}_\ell(\Omega_\ell^2 - \omega^2)} + \sum_{\ell=L+1}^{N} \frac{\phi_{i\ell}\phi_{j\ell}}{\bar{m}_\ell\Omega_\ell^2(1-\frac{\omega^2}{\Omega_\ell^2})} \tag{5}$$

Assuming that $\omega << \Omega_{\ell+1}$, equation (5) becomes

$$G_{ij}(\omega) \cong \sum_{\ell=1}^{L} \frac{\phi_{i\ell}\,\phi_{j\ell}}{\bar{m}_\ell(\Omega_\ell^2-\omega^2)} + (\underline{K}^{-1})_{ij} - \sum_{\ell=1}^{L} \frac{\phi_{i\ell}\,\phi_{j\ell}}{\bar{m}_\ell\Omega_\ell^2} \tag{6}$$

since

$$(\underline{K}^{-1})_{ij} = \sum_{\ell=1}^{N} \frac{\phi_{i\ell}\,\phi_{j\ell}}{\bar{m}_\ell\Omega_\ell^2} \tag{7}$$

It can be shown that equation (6) and the R = 1 form of equation (4) are equivalent. The last two terms in equation (6) are referred to as the residual flexibility, and take into account the static effects of the missing higher order modes. Childs[7], in a study of transient modal rotordynamic models, has recently applied the residual flexibility approach.

The present work deals with the more general receptance formed as the second degree, lambda matrix inverse

$$\underline{F}(\lambda) = (\lambda^2\underline{M}+\lambda\underline{C}+\underline{K})^{-1} \qquad \text{NXN} \tag{8}$$

where $\underline{M}$, $\underline{K}$, and $\underline{C}$ are square, generally nonsymmetric matrices. The scalar λ is in general a member of the complex number field. The matrix $\underline{F}$ will be referred to as a generalized receptance matrix. The generalized receptance is useful in vibration theory while examining rotating structures[8], structures with rotating elements, and structures with discrete dampers. This last category is gaining increasing importance due to a

growing interest in active vibration control systems[9,10].

MATHEMATICAL FORMULATION

We begin the study of equation (8) by considering the quadratic eigenvalue problem

$$(\alpha_i^2 \underline{M} + \alpha_i \underline{C} + \underline{K})\, \underline{\Lambda}_i = \underline{0} \qquad (NX1) \tag{9}$$

and the corresponding left eigenvector problem

$$(\alpha_i^2 \underline{M}^T + \alpha_i \underline{C}^T + \underline{K}^T)\, \underline{\delta}_i = \underline{0} \qquad (NX1) \tag{10}$$

These equations are expressed in first order form as[12]

$$(\alpha_k \underline{A}_1 + \underline{B}_1)\underline{\Gamma}_k = \underline{0}\ ,\ (\alpha_k \underline{A}_1^T + \underline{B}_1^T)\underline{Y}_k = \underline{0} \tag{11}$$

$$\underline{A}_1 = \left[\begin{array}{c|c} \underline{M} & \underline{0} \\ \hline \underline{0} & -\underline{K} \end{array}\right]\ ,\quad \underline{B}_1 = \left[\begin{array}{c|c} \underline{C} & \underline{K} \\ \hline \underline{K} & \underline{0} \end{array}\right] \tag{12}$$

or alternatively as

$$(\alpha_k \underline{A}_2 + \underline{B}_2)\underline{\Gamma}_k = \underline{0}\ ,\ (\alpha_k \underline{A}_2^T + \underline{B}_2^T)\underline{Y}_k = \underline{0} \tag{13}$$

$$A_2 = \left[\begin{array}{c|c} \underline{0} & \underline{M} \\ \hline \underline{M} & \underline{C} \end{array}\right]\ ,\qquad \underline{B}_2 = \left[\begin{array}{c|c} -\underline{M} & \underline{0} \\ \hline \underline{0} & \underline{K} \end{array}\right] \tag{14}$$

The modal matrices associated with equations (9) - (14) are

$$\underline{Q} = [\underline{\Lambda}_1 \mid \underline{\Lambda}_2 \mid \cdots \mid \underline{\Lambda}_{2N}]\ ,\quad \underline{V} = [\underline{\delta}_1 \mid \underline{\delta}_2 \mid \cdots \mid \underline{\delta}_{2N}] \tag{15}$$

$$\underline{\Gamma} = [\underline{\Gamma}_1 \mid \underline{\Gamma}_2 \mid \cdots \mid \underline{\Gamma}_{2N}] = \left[\begin{array}{c} \underline{Q}\ \underline{\mathrm{diag}}\ (\alpha_i) \\ \hline \underline{Q} \end{array}\right] \tag{16}$$

$$\underline{Y} = [\underline{Y}_1 \mid \underline{Y}_2 \mid \cdots \mid \underline{Y}_{2N}] = \left[\begin{array}{c} \underline{V}\ \underline{\mathrm{diag}}\ (\alpha_i) \\ \hline \underline{V} \end{array}\right] \tag{17}$$

Assume that each eigenvalue α_i is distinct with an eigenspace of dimension one. The biorthogonality conditions[13] satisfied by these eigenvectors then are

$$(\underline{Y}^T \underline{A}_1 \underline{\Gamma})_{ij} = \hat{\delta}_{ij}\, a_{1i},\ (\underline{Y}^T \underline{B}_1 \underline{\Gamma})_{ij} = -\hat{\delta}_{ij}\, a_{1i}\, \alpha_i \tag{18}$$

$$(\underline{Y}^T \underline{A}_2 \underline{\Gamma})_{ij} = \hat{\delta}_{ij}\, a_{2i}\ ,\ (\underline{Y}^T \underline{B}_2 \underline{\Gamma})_{ij} = -\hat{\delta}_{ij}\, a_{2i}\, \alpha_i \tag{19}$$

$$a_{2i} = \underline{\delta}_i^T (2\alpha_i \underline{M} + \underline{C}) \underline{\Lambda}_i\ ,\ a_{1i} = \alpha_i a_{2i} \qquad \alpha_i \neq 0 \tag{20}$$

Define the matrix pencil

$$\underline{P} = \lambda\, \underline{A}_2 + \underline{B}_2\ ,\ \lambda \neq \alpha_i \tag{21}$$

The inverse of this pencil is obtained using equations (16) - (20)

$$\underline{P}^{-1} = \left[\begin{array}{c|c} \underline{Q}\ \underline{\mathrm{diag}} \left\{\dfrac{\alpha_i^2}{a_{2i}(\lambda-\alpha_i)}\right\} \underline{V}^T & \underline{Q}\ \underline{\mathrm{diag}} \left\{\dfrac{\alpha_i}{a_{2i}(\lambda-\alpha_i)}\right\} \underline{V}^T \\ \hline \underline{Q}\ \underline{\mathrm{diag}} \left\{\dfrac{\alpha_i}{a_{2i}(\lambda-\alpha_i)}\right\} \underline{V}^T & \underline{Q}\ \underline{\mathrm{diag}} \left\{\dfrac{1}{a_{2i}(\lambda-\alpha_i)}\right\} \underline{V}^T \end{array}\right] \tag{22}$$

Substitute equations (14), (21), (22) into the identity

$$\underline{P}\ \underline{P}^{-1} = \underline{I}_{2N} \tag{23}$$

This yields the conditions

$$-\underline{M}\ \underline{Q}\ \underline{\mathrm{diag}} \left\{\frac{\alpha_i}{a_{2i}(\lambda-\alpha_i)}\right\} \underline{V}^T + \lambda\, \underline{M}\ \underline{Q}\ \underline{\mathrm{diag}} \left\{\frac{1}{a_{2i}(\lambda-\alpha_i)}\right\} \underline{V}^T = \underline{0} \tag{24}$$

$$\lambda \underline{M}\ \underline{Q}\ \underline{\mathrm{diag}} \left\{\frac{\alpha_i}{a_{2i}(\lambda-\alpha_i)}\right\} \underline{V}^T + (\lambda \underline{C} + \underline{K}) \underline{Q}\ \underline{\mathrm{diag}} \left\{\frac{1}{a_{2i}(\lambda-\alpha_i)}\right\} \underline{V}^T = \underline{I}_N \tag{25}$$

Multiplying equation (24) by λ and adding the results to (25) provides the spectral representation of the generalized receptance matrix

$$\underline{F}(\lambda) = \sum_{j=1}^{2N} \frac{\underline{\Lambda}_j \underline{\delta}_j^T}{a_{2j}(\lambda-\alpha_j)} \tag{26}$$

When the structure is constrained and unable to undergo a rigid body displacement in any vibratory coordinate direction, all 2N eigenvalues (α_i) are non-zero. In the limit as λ approaches zero equation (8) and (26) imply

$$\underline{K}^{-1} = -\sum_{j=1}^{2N} \frac{\underline{\Lambda}_j \underline{\delta}_j^T}{a_{2j} \alpha_j} \tag{27}$$

Consider the following finite expansion

$$\frac{1}{\lambda-\alpha_j} = -\frac{1}{\alpha_j} - \frac{\lambda}{\alpha_j^2} - \frac{\lambda^2}{\alpha_j^3} - \frac{\lambda^3}{\alpha_j^4} - \ldots - \frac{\lambda^{R-1}}{\alpha_j^R} + \frac{\lambda^R}{\alpha_j^R(\lambda-\alpha_j)} \tag{28}$$

R = 1, 2, ..., R finite

Substitute equation (28) into (26)

$$\underline{F}(\lambda) = -\sum_{j=1}^{2N} \frac{\underline{\Lambda}_j \underline{\delta}_j^T}{a_{2j} \alpha_j} - \lambda \sum_{j=1}^{2N} \frac{\underline{\Lambda}_j \underline{\delta}_j^T}{a_{2j} \alpha_j^2} - \lambda^2 \sum_{j=1}^{2N} \frac{\underline{\Lambda}_j \underline{\delta}_j^T}{a_{2j} \alpha_j^3}$$

$$-\lambda^3 \sum_{j=1}^{2N} \frac{\underline{\Lambda}_j \underline{\delta}_j^T}{a_{2j} \alpha_j^4} - \ldots - \lambda^{R-1} \sum_{j=1}^{2N} \frac{\underline{\Lambda}_j \underline{\delta}_j^T}{a_{2j} \alpha_j^R} + \lambda^R \sum_{j=1}^{2N} \frac{\underline{\Lambda}_j \underline{\delta}_j^T}{a_{2j} \alpha_j^R(\lambda-\alpha_j)} \tag{29}$$

It follows from equation (27) that the first term in this series equals the inverse of the stiffness matrix. Likewise, the first R terms of

(29) may be expressed in terms of the $\underline{M}$, $\underline{K}$ and $\underline{C}$ property matrices. These expressions are obtained by equating the spectral representation for

$$\underline{B}_1^{-1} \left(\underline{A}_1 \underline{B}_1^{-1}\right)^{R-2} \qquad R \geq 2 \qquad (30)$$

to its representation as defined by equations (12). The spectral representations for $\underline{A}_1^{-1}$ and $\underline{B}_1^{-1}$ are derived using equations (18) - (20).

The spectral representations for $\underline{A}_1^{-1}$ and $\underline{B}_1^{-1}$ are derived using equations (18) - (20).

$$\underline{A}_1^{-1} = \underline{\Gamma}\ \underline{\text{diag}} \left\{\frac{1}{\alpha_j\ a_{2j}}\right\} \underline{Y}^T = \sum_{j=1}^{2N} \frac{\left[\begin{array}{c|c} \alpha_j^2\ \underline{\Lambda}_j\ \underline{\delta}_j^T & \alpha_j\ \underline{\Lambda}_j\ \underline{\delta}_j^T \\ \hline \alpha_j\ \underline{\Lambda}_j\ \underline{\delta}_j^T & \underline{\Lambda}_j\ \underline{\delta}_j^T \end{array}\right]}{\alpha_j\ a_{2j}} \qquad (31)$$

$$\underline{B}_1 = -\ \underline{\Gamma}\ \underline{\text{diag}} \left\{\frac{1}{\alpha_j^2\ a_{2j}}\right\} \underline{Y}^T = -\sum_{j=1}^{2N} \frac{\left[\begin{array}{c|c} \alpha_j^2\ \underline{\Lambda}_j\ \underline{\delta}_j^T & \alpha_j\ \underline{\Lambda}_j\ \underline{\delta}_j^T \\ \hline \alpha_j\ \underline{\Lambda}_j\ \underline{\delta}_j^T & \underline{\Lambda}_j\ \underline{\delta}_j^T \end{array}\right]}{\alpha_j^2\ a_{2j}} \qquad (32)$$

In deriving equivalent expression in terms of $\underline{M}$, $\underline{K}$, and $\underline{C}$ for the summations in equation (29), only the "2,2" submatrix of equation (30) need be examined. The hybrid spectral-property matrix representations for the generalized receptance matrix for R equal to 1 through 4 are

$\underline{R = 0}$

$$\underline{F}(\lambda) = \sum_{j=1}^{2N} \frac{\underline{\Lambda}_j\ \underline{\delta}_j^T}{a_{2j}(\lambda - \alpha_j)} \qquad (33)$$

$\underline{R = 1, \alpha_j \neq 0}$

$$\underline{F}(\lambda) = \underline{K}^{-1} + \lambda \sum_{j=1}^{2N} \frac{\underline{\Lambda}_j \underline{\delta}_j^T}{a_{2j} \alpha_j (\lambda - \alpha_j)} \tag{34}$$

$\underline{R = 2, \alpha_j \neq 0}$

$$\underline{F}(\lambda) = \{\underline{I}_N - \lambda \underline{K}^{-1} \underline{C}\} \underline{K}^{-1} + \lambda^2 \sum_{j=1}^{2N} \frac{\underline{\Lambda}_j \underline{\delta}_j^T}{a_{2j} \alpha_j^2 (\lambda - \alpha_j)} \tag{35}$$

$\underline{R = 3, \alpha_j \neq 0}$

$$\underline{F}(\lambda) = \{\underline{I}_N - \lambda \underline{K}^{-1} \underline{C} - \lambda^2 \{\underline{K}^{-1} \underline{M} \underline{K}^{-1} \underline{C} \underline{K}^{-1} \underline{C}\}\} \underline{K}^{-1}$$

$$+ \lambda^3 \sum_{j=1}^{2N} \frac{\underline{\Lambda}_j \underline{\delta}_j^T}{a_{2j} \alpha_j^3 (\lambda - \alpha_j)} \tag{36}$$

$\underline{R = 4, \alpha_j \neq 0}$

$$\underline{F}(\lambda) = \{\underline{I}_N - \lambda \underline{K}^{-1} \underline{C} - \lambda^2 \{\underline{K}^{-1} \underline{M} \underline{K}^{-1} \underline{C} \underline{K}^{-1} \underline{C}\}$$

$$-\lambda^3 \{-\underline{K}^{-1} \underline{C} \underline{K}^{-1} \underline{M} \underline{K}^{-1} \underline{M} \underline{K}^{-1} \underline{C} + \underline{K}^{-1} \underline{C} \underline{K}^{-1} \underline{C} \underline{K}^{-1} \underline{C}\}\} \underline{K}^{-1}$$

$$+\lambda^4 \sum_{j=1}^{2N} \frac{\underline{\Lambda}_j \underline{\delta}_j^T}{a_{2j} \alpha_j^4 (\lambda - \alpha_j)} \tag{37}$$

These equations (34) - (37) each have the form

$\underline{F}(\lambda)$ = Property Matrix Contribution + Modal Contribution

For large systems the modal contribution is normally incomplete since the higher order modes are not calculated. In practice the summations in equations (33) - (37) only extend over the 2L lowest modes (including complex conjugate modes) where L typically is in the range of equation (3). If the eigensystem employed is complete all of these formulas will yield the exact receptance matrix defined by equation (8).

IMPROVING THE COMPUTATIONAL ACCURACY

The integer R in equations (29) and (33) - (37) can be referred to as a "higher mode factor." To investigate why increasing R reduces the importance of the higher mode contributions to the summations in equations (33) - (37) consider the ratio

$$r_n = \left| \frac{\text{coefficient of } \underline{\Lambda}_n \, \underline{\delta}_n^T}{\text{coefficient of } \underline{\Lambda}_1 \, \underline{\delta}_1^T} \right| = \left| \frac{\alpha_1}{\alpha_n} \right|^R \cdot \left| \frac{a_{21}(\lambda - \alpha_1)}{a_{2n}(\lambda - \alpha_n)} \right| \tag{38}$$

Arrange the eigensolutions according to

$$|\alpha_1| \leq |\alpha_2| \leq |\alpha_3| \leq |\alpha_4| \cdots |\alpha_{2N-1}| \leq |\alpha_{2N}| \tag{39}$$

where the equality holds only if $\alpha_{j+1} = \bar{\alpha}_j$. This implies that

$$\left| \frac{\alpha_1}{\alpha_n} \right| < 1 , \quad 3 \leq n \leq 2N \tag{40}$$

Consequently equation (38) shows that as R increases, r_n decreases for n greater then three (3). Hence the higher mode contributions to the summations in equations (33) - (37) are increasingly less important for larger values of R. Since these modes are not usually calculated in the analysis of large structure models the accuracy of the elements of $\underline{F}(\lambda)$ will improve as R increases.

Consider the contribution of the mth mode to the receptance matrices in equations (33) - (37), i.e.,

$\underline{R = 0}$

$$\frac{\underline{\Lambda}_m \, \underline{\delta}_m^T}{a_{2m}(\lambda - \alpha_m)} = \underline{\varepsilon}_m$$

$\underline{R = 1}$

$$\frac{\lambda}{\alpha_m} \frac{\underline{\Lambda}_m \, \underline{\delta}_m^T}{a_{2m}(\lambda - \alpha_m)} = \frac{\lambda}{\alpha_m} \underline{\varepsilon}_m$$

R = 2

$$\frac{\lambda^2}{\alpha_m^2}\ \frac{\underline{\Delta}_m\ \underline{\delta}_m^T}{a_{2m}(\lambda-\alpha_m)} = \left(\frac{\lambda}{\alpha_m}\right)^2\ \underline{\varepsilon}_m$$

etc.

Clearly the magnitude of the contribution from the mth mode will decrease as R increases if

$$\left|\frac{\lambda}{\alpha_m}\right| < 1$$

This shows that if only q eigensolutions are utilized in equations (33) - (37) then

$$\left|\frac{\lambda}{\alpha_q}\right| < 1$$

is a necessary condition for the R = 1,2,3... receptance formulas to produce more accurate results than the R = 0 formula. Therefore, the ($R \leq 1$) receptance formulas for $\underline{F}(\lambda)$ should only be used if $|\lambda| < |\alpha_q|$.

APPLICATION TO SPECIAL SYSTEM TYPES

The generalized receptance formulas given by equations (29), and (33) - (37) can be simplified for several common types of vibrational systems. Gyroscopic systems satisfy

$$\underline{M} > 0,\ \underline{K} \geq 0,\ \underline{C}^T = -\underline{C} \tag{41}$$

These frequently occur in models of rotating structures[14,15]. For this system type[13]

$$\underline{\delta}_j = \overline{\underline{\Lambda}}_j,\ Re(\alpha_j) = 0,\ Re(a_{2j}) = 0 \tag{42}$$

The eigenvectors of general, symmetric systems

$$\underline{M} > 0,\ \underline{C} \geq 0,\ \underline{K} \geq 0 \tag{43}$$

satisfy

$$\underline{\delta}_j = \underline{\Lambda}_j \tag{44}$$

Proportionately damped vibratory systems are defined by

$$\underline{M} > 0,\ \underline{K} \geq 0,\ \underline{\Phi}_i^T\ \underline{C}\ \underline{\Phi}_j = \hat{\delta}_{ij}\ 2\zeta_i\ \Omega_i\ \overline{m_i} \tag{45}$$

In this case equations (29), (33) - (37) reduce to

$$\underline{R = 0}$$

$$\underline{F}(\lambda) = \sum_{k=1}^{N} \frac{\underline{\Phi}_k\ \underline{\Phi}_k^T}{\overline{m}_k\ (\lambda^2 + 2\zeta_k\ \Omega_k\ \lambda + \Omega_k^2)} \tag{46}$$

$$\underline{R = 1, 2, \ldots}$$

$$\underline{F}\ (\lambda) = \{\underline{I}_N + \underline{K}^{-1}(-\lambda^2\underline{M} - \lambda\ \underline{C}) + \ldots$$

$$+ (\underline{K}^{-1}\ (-\lambda^2\underline{M} - \lambda\ \underline{C}))^{R-1}\}\ \underline{K}^{-1}$$

$$+ (-\lambda^2)^R \sum_{k=1}^{N} \frac{(\lambda + 2\zeta_k\Omega_k)^R\ \underline{\Phi}_k\ \underline{\Phi}_k^T}{\overline{m}_k(\lambda\ \Omega_k^2)^R\ (\lambda^2 + 2\zeta_k\Omega_k\lambda + \Omega_k^2)} \tag{47}$$

USE OF THE GENERAL RECEPTANCE MATRIX

Use of the receptance matrix derived here typically requires that only a small submatrix of $\underline{F}$ be evaluated, i.e.,

$$\underline{f}(\lambda) = \underline{F}\begin{pmatrix} k_1 & k_2 & \cdots & k_s \\ j_1 & j_2 & \cdots & j_p \end{pmatrix} = \begin{bmatrix} F_{k_1 j_1} & F_{k_1 j_2} & \cdots & F_{k_1 j_p} \\ F_{k_2 j_1} & F_{k_2 j_2} & \cdots & F_{k_2 j_p} \\ & & \cdots & \\ F_{k_s j_1} & F_{k_s j_2} & \cdots & F_{k_s j_p} \end{bmatrix}$$

$$s << N,\ p << N \tag{48}$$

Consider a system model where

$$(k_1\ k_2\ k_3\ k_4) = (j_1\ j_2\ j_3\ j_4) = (6\ 7\ 8\ 9)$$

The R = 1 formula from equation (34) implies

$$\underline{f}(\lambda) = \begin{bmatrix} \underline{K}_{66}^{-1} & \underline{K}_{67}^{-1} & \underline{K}_{68}^{-1} & \underline{K}_{69}^{-1} \\ \underline{K}_{76}^{-1} & \underline{K}_{77}^{-1} & \underline{K}_{78}^{-1} & \underline{K}_{79}^{-1} \\ \underline{K}_{86}^{-1} & \underline{K}_{87}^{-1} & \underline{K}_{88}^{-1} & \underline{K}_{89}^{-1} \\ \underline{K}_{96}^{-1} & \underline{K}_{97}^{-1} & \underline{K}_{98}^{-1} & \underline{K}_{99}^{-1} \end{bmatrix} + \lambda \begin{bmatrix} \Delta_{66}^1 & \Delta_{67}^1 & \Delta_{68}^1 & \Delta_{69}^1 \\ \Delta_{76}^1 & \Delta_{77}^1 & \Delta_{78}^1 & \Delta_{79}^1 \\ \Delta_{86}^1 & \Delta_{87}^1 & \Delta_{88}^1 & \Delta_{89}^1 \\ \Delta_{96}^1 & \Delta_{97}^1 & \Delta_{98}^1 & \Delta_{99}^1 \end{bmatrix}$$

$$\Delta_{ij}^1 = \sum_{\ell=1}^{N} \frac{\Lambda_{i\ell}\ \delta_{j\ell}}{a_{2\ell}\ \alpha_\ell(\lambda - \alpha_\ell)} \tag{49}$$

The R = 2 formula from equation (35) becomes

$$\underline{f}(\lambda) = \begin{bmatrix} \underline{K}_{66}^{-1} & \underline{K}_{67}^{-1} & \underline{K}_{68}^{-1} & \underline{K}_{69}^{-1} \\ \underline{K}_{76}^{-1} & \underline{K}_{77}^{-1} & \underline{K}_{78}^{-1} & \underline{K}_{79}^{-1} \\ \underline{K}_{86}^{-1} & \underline{K}_{87}^{-1} & \underline{K}_{88}^{-1} & \underline{K}_{89}^{-1} \\ \underline{K}_{96}^{-1} & \underline{K}_{97}^{-1} & \underline{K}_{98}^{-1} & K_{99}^{-1} \end{bmatrix}$$

$$-\lambda \begin{bmatrix} \tilde{\underline{K}}_6^{-1} \\ \hline \underline{K}_7^{-1} \\ \hline \tilde{\underline{K}}_8^{-1} \\ \hline \tilde{\underline{K}}_9^{-1} \end{bmatrix} \begin{bmatrix} \underline{C} \end{bmatrix} \left[\begin{array}{c|c|c|c} \underline{K}_6^{-1} & \underline{K}_7^{-1} & \underline{K}_8^{-1} & \underline{K}_9^{-1} \end{array}\right] + \lambda^2 \begin{bmatrix} \Delta_{66}^2 & \Delta_{67}^2 & \Delta_{68}^2 & \Delta_{69}^2 \\ \Delta_{76}^2 & \Delta_{77}^2 & \Delta_{78}^2 & \Delta_{79}^2 \\ \Delta_{86}^2 & \Delta_{87}^2 & \Delta_{88}^2 & \Delta_{89}^2 \\ \Delta_{96}^2 & \Delta_{97}^2 & \Delta_{98}^2 & \Delta_{99}^2 \end{bmatrix}$$

where

$$\Delta_{ij}^2 = \sum_{\ell=1}^{N} \frac{\Lambda_{i\ell}\ \delta_{j\ell}}{a_{2\ell}\ \alpha_\ell^2(\lambda - \alpha_\ell)} \tag{50}$$

and $\underline{\tilde{K}}_n^{-1}$ and $\underline{K}_n^{-1}$ are the nth row and column of $\underline{K}^{-1}$, respectively, Similar formulas for R greater than 2 are also easily derived. Note that the property matrix - submatrices in equations (49) and (50) need be evaluated and stored only once. These submatrices may then be retrieved to evaluate $\underline{f}(\lambda)$ for any value of λ.

APPLICATION OF THE RECEPTANCE FORMULA

A useful application of receptance formulas is in dynamic reanalysis. Suppose that modifications to a structure result in the formation of the new mass, damping, and stiffness matrices

$$\underline{\tilde{M}} = \underline{M} + \Delta\underline{M} \qquad (N \times N) \tag{51}$$

$$\underline{\tilde{C}} = \underline{C} + \Delta\underline{C} \tag{52}$$

$$\underline{\tilde{K}} = \underline{K} + \Delta\underline{K} \tag{53}$$

The quadratic matrix eigenvalue problem for the modified system becomes

$$(\lambda_n^2(\underline{M} + \Delta\underline{M}) + \lambda_n\ (\underline{C} + \Delta\underline{C}) + (\underline{K} + \Delta\underline{K}))\underline{\psi}_n = \underline{0} \tag{54}$$

From equation (54) define the modification matrix

$$\underline{S}(\lambda) = \lambda^2\ \Delta\underline{M} + \ \Delta\underline{C} + \Delta\underline{K} \tag{55}$$

Suppose that the modification involves relatively few elements in the property matrices, in other words, $\Delta\underline{M}$ etc., is very sparse. The sparseness of the modification is quantified by considering the non-null rows (or columns) of $\underline{S}(\lambda)$. These row numbers are entered in the set

$$\underline{J} = (j_1\ j_2\ \cdots\ j_p)^T \qquad p \times 1 \tag{56}$$

A local modification to the structure is defined as a modification with a p/N ratio much less than 1. Appendix II demonstrates that a form of the modified system's characteristic equation is

$$g(\lambda_n) = \text{determinant } (\underline{I}_p + \hat{\underline{S}}(\lambda_n)\ \hat{\underline{F}}(\lambda_n)) = 0 \qquad (57)$$

where

λ_n: zeroes of $g(\lambda)$ (eigenvalues of the modified structure)

$$\hat{\underline{S}}(\lambda) = \underline{S} \begin{pmatrix} j_1\ j_2 \cdots j_p \\ j_1\ j_2 \cdots j_p \end{pmatrix} \qquad pxp \qquad (58)$$

= condensed modification matrix

$$\hat{\underline{F}}(\lambda) = \underline{F} \begin{pmatrix} j_1\ j_2 \cdots j_p \\ j_1\ j_2 \cdots j_p \end{pmatrix} \qquad pxp \qquad (59)$$

= condensed receptance matrix

$\underline{I}_p$ = unit matrix of order p

Equations (58) and (59) utilize the notation of equation (48). As seen above, the generalized receptance matrix enters into the reanalysis procedure through the matrix $\hat{\underline{F}}(\lambda)$. The zeroes of $g(\lambda)$ are obtained iteratively, using either the Newton Raphson or Muller approach. During the iteration the condensed receptance matrix $\hat{\underline{F}}(\lambda)$ must be repeatedly evaluated for each guess of λ. In addition a search for an optimal structural design requires computation of the eigenvalues for each trial design. These designs result in the formation of $\Delta\underline{M}$, $\Delta\underline{K}$, and $\Delta\underline{C}$ modification matrices and consequently in the formation of condensed effective modification matrices $\hat{\underline{S}}$. This emphasizes the requirement for computation of $\hat{\underline{F}}(\lambda)$ for many values of λ. The eigensolution reanalysis procedure is therefore a reasonable application for the approximate receptance formulas previously derived.

NUMERICAL EXAMPLE

The uniform beam model in Figure 1 can be used to demonstrate the accuracy and efficiency of the receptance approach to an eigenvalue reanalysis

as outlined in Appendix II. This NASA model is a crude continuum representation of a large space truss. The objective is to determine the optimal location for an "active" damper (c) to dampen transient vibrations resulting from maneuvers, dockings, and other dynamic loadings.

Classical root locus plots are constructed, varying the damper strength (c) from 0.1 to 4.0 lb. sec. $in.^{-1}$ ($0.0175 \leq c \leq 0.70$ kN sec. m^{-1}), in 0.1 lb. sec. $in.^{-1}$ (0.0175 kN sec. m^{-1}) increments. For each damper value the lower portion of the beam's eigensystem, as determined from the imaginary part of the eigenvalues, is computed. These plots are generated for various damper locations and consequently may mandate the computation of the partial eigensystem for hundreds of cases.

In this particular example the system has 50 dof, i.e., N = 50 and the generalized displacement vector is

$$\underline{\Lambda} = (X_1 \quad \theta_1 \quad X_2 \quad \theta_2 \cdots X_{25} \quad \theta_{25})^T$$

Suppose a damper is added at node 11, then $p = 1$ and $\underline{J} = (j_1) = (21)$. The general equations of the previous section reduce to

$$\hat{\underline{S}}(\lambda) = \lambda^2 \hat{\Delta\underline{M}} + \lambda \hat{\Delta\underline{C}} + \hat{\Delta\underline{K}} = \lambda \hat{\Delta C}$$

since

$$\hat{\Delta\underline{M}} = \hat{\Delta\underline{K}} = \underline{0}, \quad \hat{\Delta\underline{C}} = (\Delta\underline{C})_{21,21} = c$$

Also, for this case

$$\hat{\underline{F}}(\lambda) = (\underline{F})_{21,21} = F_{21,21}$$

For this case, equation (75) of Appendix II yields the following form for the modified system's characteristic equation

$$g(\lambda) = 1 + \lambda c \, F_{21,21}(\lambda) = 0$$

Since the initial system is undamped and is described by symmetric $\underline{M}$ and $\underline{K}$ matrices, the receptance $F_{21,21}$ is computed, for each guess of λ, by using equations (46) and (47)

R = 0

$$F_{21,21}(\lambda) \cong \sum_{k=1}^{L} \frac{\phi_{21,k}^2}{\bar{m}_k(\lambda^2+\Omega_k^2)}$$

R = 1

$$F_{21,21}(\lambda) \cong (\underline{K}^{-1})_{21,21} - \lambda^2 \sum_{k=1}^{L} \frac{\phi_{21,k}^2}{\bar{m}_k\Omega_k^2(\lambda^2+\Omega_k^2)}$$

where typically L < < N, i.e. an incomplete eigensystem is used, and ϕ_{ij}, $\bar{m}_j$ and Ω_j are modal properties of the undamped system. The zeroes of $g(\lambda)$ are eigenvalues of the damped system, and are obtained with Muller's method. Root locus plots for a damper at node 11, are shown in Figure 2. Optimum damper and optimum damping ratio (ζ) values are listed for each mode. Twelve (L = 12) original system modes were employed in the receptance formulas ($F_{21,21}$), above. Figure 2 shows results for both the R = 0 and R = 1 receptance formulas. The eigenvalue solver EISPACK[16] was used to judge the accuracy of the approximate reanalysis results. EISPACK conditions the input matrix ($\underline{A}_2^{-1}\ \underline{B}_2$, see equation (13)) with subroutine BALANC, reduces the conditioned matrix to Hessenberg form[17] with subroutine ELMHES, and then computes the eigenvalues of this similar matrix with the QR algorithm, utilizing subroutine HQR. Table I shows the maximum percent error for each of the eight (8) lowest modes, for damper values in the set {0.1, 0.2, 0.4, 0.7, 1.1, 1.2, 1.4, 1.5, 4.0} lb sec. $in.^{-1}$ (0.0175, 0.035, 0.07, 0.1925, 0.21, 0.245, 0.2625, 0.70 kN sec. m^{-1}). Note by comparison with Figure 2 that this set of (c) values encompasses all of the optimum dampers, and the minimum and maximum damper values. The results clearly show

that the computation of the receptances by the R = 1 formula provides substantially improved eigenvalue reanalysis accuracy, as compared to the R = 0 receptance computation results. Table II shows a computation time efficiency comparison for the exact (EISPACK) and approximate (reanalysis) methods. A large computer time reduction is seen to result with the approximate reanalysis method. For instance, the average reanalysis time per value of C for the R = 1 approach is 0.4725 cp. secs.. while the same computation requires 37.6 cp. secs. with EISPACK. This represents a computation time reduction by a factor of 79.6.

This numerical example only includes eigenvalue computations since the root locus plots do not require eigenvectors.

CONCLUDING REMARKS

A representation for the approximate calculation of the receptance elements for a general second degree, square lambda matrix has been presented. The computational efficiency of solving for a submatrix of the receptance is increased, in particular when the submatrix is evaluated for many distinct values of λ. The submatrices are useful in eigensolution reanalysis, design, or component synthesis schemes, as well as in forced harmonic response analyses. The accuracy of the receptances are shown to be improved relative to a classical spectral representation when employing only an incomplete eigensystem. Previous receptance formulas were shown to be special cases of the general results derived. These prior works apply only to undamped, nonrotating structures, with symmetrical property matrices.

It was assumed in equations (18) - (20) that the eigenvalues of the structural model are all distinct. In addition, in equation (63) the

assumption was made that no eigenvalue is left invariant during a structural modification. Because of the lack of ideal symmetry characteristics, both of these conditions are satisfied in many structural models. These limitations on the applicability of the formulation are subjects of ongoing research.

APPENDIX I: NOTATION

General Notation

$\underline{\ }$ = matrix-vector quantity

$\underline{A}^{-1}$ = inverse of matrix $\underline{A}$

$\underline{A}>0$ = $\underline{A}$ symmetric, positive definite

$\underline{A} \geq 0$ = $\underline{A}$ symmetric, positive semi-definite

$\underline{\text{diag}}\ (\ )$ = diagonal matrix

$\overline{(\ )}$ = complex conjugate

$\hat{\delta}_{ij}$ = Kronecker delta

Re ( ) = real part of

$i = \sqrt{-1}$

$\underline{A}^{T}$ = transpose of matrix $\underline{A}$

$\underline{A}_1, \underline{B}_1, \underline{A}_2, \underline{B}_2$ = first order forms of system property matrices

a_{1i}, a_{2i} = modal normalization constants

$\underline{C}$ = damping matrix NXN

$\underline{f}(\lambda)$ = a submatrix of $\underline{F}$, equation (48)

$\underline{F}$ = generalized receptance matrix

$\underline{G}$ = classical receptance or dynamic flexibility influence coefficient matrix

$\underline{K}$ = stiffness matrix NXN

L = number of system eigensolutions available for use in a spectral type representation of $\underline{F}$ or $\underline{G}$

$\underline{M}$ = mass matrix NXN

$\bar{m}_\ell$ = modal normalization constant

$$\bar{m}_\ell = \Phi_\ell^T \underline{M} \Phi_\ell$$

N = total number of system model degrees of freedom

$\underline{P}$ = 2N X 2N matrix pencil

$\underline{Q}$ = N X 2N right eigenvector modal matrix

R = receptance higher mode factor

$\underline{V}$ = N X 2N left eigenvector modal matrix

α_i = ith complex eigenvalue of a general vibratory system

$\underline{\Gamma}$ = 2N X 2N modal matrix of right eigenvectors from a first order system formulation

$\underline{\Gamma}_k$ = kth column of $\underline{\Gamma}$

$\underline{Y}$ = 2N x 2N modal matrix of left eigenvectors from a first order system formulation

$\underline{Y}_k$ = kth column of $\underline{Y}$

$\underline{\delta}_i$ = ith modal damping constant for a proportionately damped system, with symmetric $\underline{M}$, $\underline{K}$, $\underline{C}$

ζ_i = ith modal damping constant for a symmetric, proportionately damped system

$\underline{\Lambda}_i$ = ith complex right eigenvector of a general vibratory system

λ = a complex scalar

$\underline{\xi}_n$ = nth generalized coordinate vector for a modified structural system

$\underline{\xi}_{in}$ = ith element of $\underline{\xi}_n$

$\underline{\Phi}_j$ = jth eigenvector of an undamped vibratory system model, with symmetric $\underline{M}$, $\underline{K}$

Φ_{ij} = ith component of $\underline{\Phi}_j$

$\underline{\psi}_n$ = nth right eigenvector of a modified structural system

ω = forcing frequency

Ω_j = jth eigenvalue of an undamped vibratory system model, with symmetric $\underline{M}$, $\underline{K}$

APPENDIX II: REANALYSIS THEORY

The free vibration problem for a modified structural system is expressed by

$$(\lambda_n^2(\underline{M} + \Delta\underline{M}) + \lambda_n(\underline{C} + \Delta\underline{C}) + (\underline{K} + \Delta\underline{K}))\underline{\psi}_n = \underline{0} \qquad (N \times 1) \qquad (60)$$

An effective modification matrix may then be defined as

$$\underline{S}(\lambda) = \lambda^2\Delta\underline{M} + \lambda\Delta\underline{C} + \Delta\underline{K} \qquad (61)$$

Assume that matrix $\underline{S}$ is very sparse. This sparsity is quantified through the integer set

$$\underline{J} = (j_1 \quad j_2 \quad \cdots \quad j_p)^T \qquad (p \times 1) \qquad (62)$$

where j_i are the non-null row (or column) numbers of $\underline{S}$. A local modification to the structure is defined as one which has a P/N ratio much less than 1. Define the generalized receptance matrix as

$$\underline{F}(\lambda) = (\lambda^2\underline{M} + \lambda\underline{C} + \underline{K})^{-1} \qquad (63)$$

which exists as long as λ is not an eigenvalue of the unmodified system. Substitute the change of basis

$$\underline{\psi}_n = \underline{F}(\lambda_n)\underline{\xi}_n(\lambda_n) \qquad (N \times 1) \qquad (64)$$

into equation (60). This yields

$$(\underline{I}_n + \underline{S}(\lambda_n)\underline{F}(\lambda_n))\underline{\xi}_n = \underline{0} \qquad (N \times 1) \qquad (65)$$

By the definition in equation (62) it follows that

$$\xi_{kn} = 0, \qquad k \notin (j_1 \quad j_2 \quad \cdots \quad j_p) \qquad (66)$$

and

$$\xi_{j_r n} + \tilde{\underline{S}}_{j_r}\,\bar{\underline{F}}\,\hat{\underline{\xi}}_n = 0, \qquad r \in (1, 2, \ldots p) \qquad (67)$$

where

$$\tilde{\underline{S}}_{j_r} = j_r \text{ row of } \underline{S} \tag{68}$$

$$\tilde{\underline{F}} = (\underline{F})\begin{pmatrix} 1 & 2 & \cdots & N \\ j_1 & j_2 & \cdots & j_p \end{pmatrix} \qquad (N \times p) \tag{69}$$

$$\hat{\underline{\xi}}_n = (\xi_{j_1 n} \quad \xi_{j_2 n} \quad \cdots \quad \xi_{j_p n})^T \tag{70}$$

Taking advantage of the sparsity of the columns of $\underline{S}$, equation (67) condenses further to

$$(\underline{I}_p + \hat{\underline{S}}(\lambda_n)\hat{\underline{F}}(\lambda_n))\hat{\underline{\xi}}_n = \underline{0} \qquad (p \times 1) \tag{71}$$

where (in the notation of equation (48))

$$\hat{\underline{S}} = (\underline{S})\begin{pmatrix} j_1 & j_2 & \cdots & j_p \\ j_1 & j_2 & \cdots & j_p \end{pmatrix}$$

$$= \lambda^2(\Delta\underline{M})\begin{pmatrix} j_1 & j_2 & \cdots & j_p \\ j_1 & j_2 & \cdots & j_p \end{pmatrix} + \lambda(\Delta\underline{C})\begin{pmatrix} j_1 & j_2 & \cdots & j_p \\ j_1 & j_2 & \cdots & j_p \end{pmatrix} + (\Delta\underline{K})\begin{pmatrix} j_1 & j_2 & \cdots & j_p \\ j_1 & j_2 & \cdots & j_p \end{pmatrix}$$

$$(p \times p) \tag{72}$$

$$\hat{\underline{F}} = (\underline{F})\begin{pmatrix} j_1 & j_2 & \cdots & j_p \\ j_1 & j_2 & \cdots & j_p \end{pmatrix} \qquad (p \times p) \tag{73}$$

Equation (71), accompanied by

$$\underline{\psi}_n = \tilde{\underline{F}}\,\hat{\underline{\xi}}_n \qquad (N \times 1) \tag{74}$$

represents the final condensed form of the quadratic eigenvalue problem (60) for the modified system. The rank of the coefficient matrix in equation (71) must be less than p, for a nontrivial null space to exist. This implies that

$$g(\lambda_n) = \det(\underline{I}_p + \hat{\underline{S}}(\lambda_n)\hat{\underline{F}}(\lambda_n)) = 0 \tag{75}$$

The zeroes of $g(\lambda_n)$ are eigenvalues of the modified structural system. Consequently, $g(\lambda)$ is a form of the modified system's characteristic equation.

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Table I Accuracy Comparison: Approximate (Reanalysis) Versus Exact (EISPACK) Eigenvalues for a Damped Beam*

	R = 0 Re(λ) Max % error	R = 1 Re(λ) Max % error	R = 0 Im(λ) Max % error	R = 1 Im(λ) Max % error
λ_1	-0.008	0.000	0.000	0.000
λ_2	-0.016	0.000	0.000	0.000
λ_3	-0.073	0.000	0.016	0.000
λ_4	-0.458	0.000	0.042	0.000
λ_5	-1.440	-0.005	0.125	0.000
λ_6	-2.17	-0.024	0.248	0.003
λ_7	-9.83	-0.318	0.016	0.000
λ_8	-7.82	-0.387	0.804	0.038

*Data shown is the maximum percent error with C ε {0.1, 0.2, 0.4, 0.7, 1.1, 1.2, 1.4, 1.5, 4.0} lb. sec. $in.^{-1}$ (0.0175, 0.035, 0.07, 0.1925, 0.21, 0.245, 0.2625, 0.70) kN sec. m^{-1}

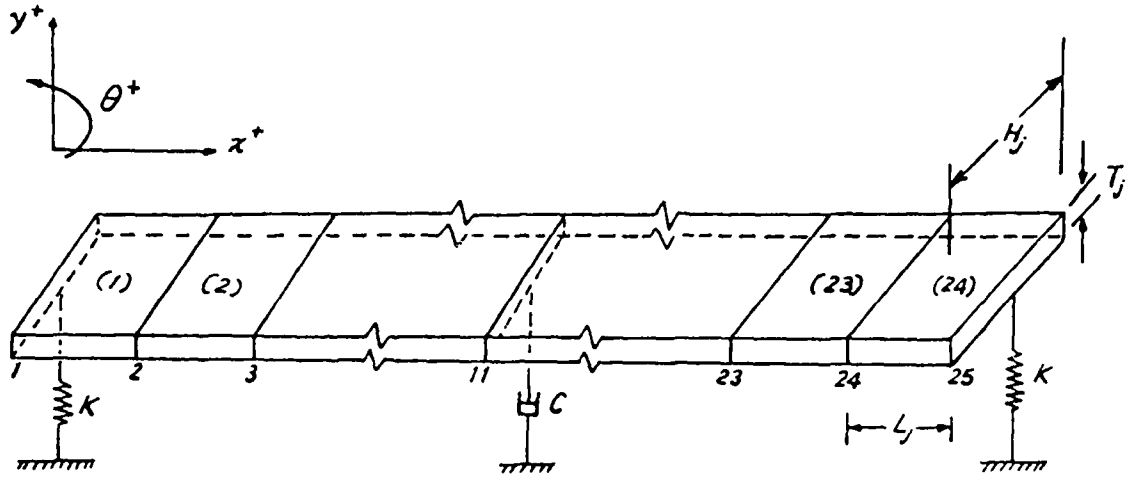
Table II Computation Time Comparison: Approximate (Reanalysis) Versus Exact (EISPACK) Approaches

(a) Formation of $\underline{K}$, $\underline{M}$ and computation of original system eigensolutions: 19.46 cp. secs.

(b) R = 0 reanalysis for c ε {0.1, 0.2, ... 4.0}, 40 values of c and 10 λ_i per value: 16.8 cp. secs., average: 0.42 sec/damper value

(c) R = 1 reanalysis for c ε {0.1, 0.2, ..., 4.0}, 40 values of c and 10 λ_i per value: 18.9 cp. secs., average: 0.4725 sec/damper value

(d) EISPACK solution for 9 values of c and 10 λ_i per value c ε {0.1, 0.2, 0.4, 0.7, 1.1, 1.2, 1.4, 1.5, 4.0}: 338.0 cp. sec., average: 37.5556 sec/damper value



K = 0.10 lb/in (17.51 N/m)

H_j = 6.0 in. (15.24 cm)

T_j = 0.1875 in. (0.47625 cm)

L_j = 6.0 in. (15.24 cm)

L = 144 in. (3.6576 m)

ρg = 0.0967 lb/in^3 (0.02596 N/cm^3)

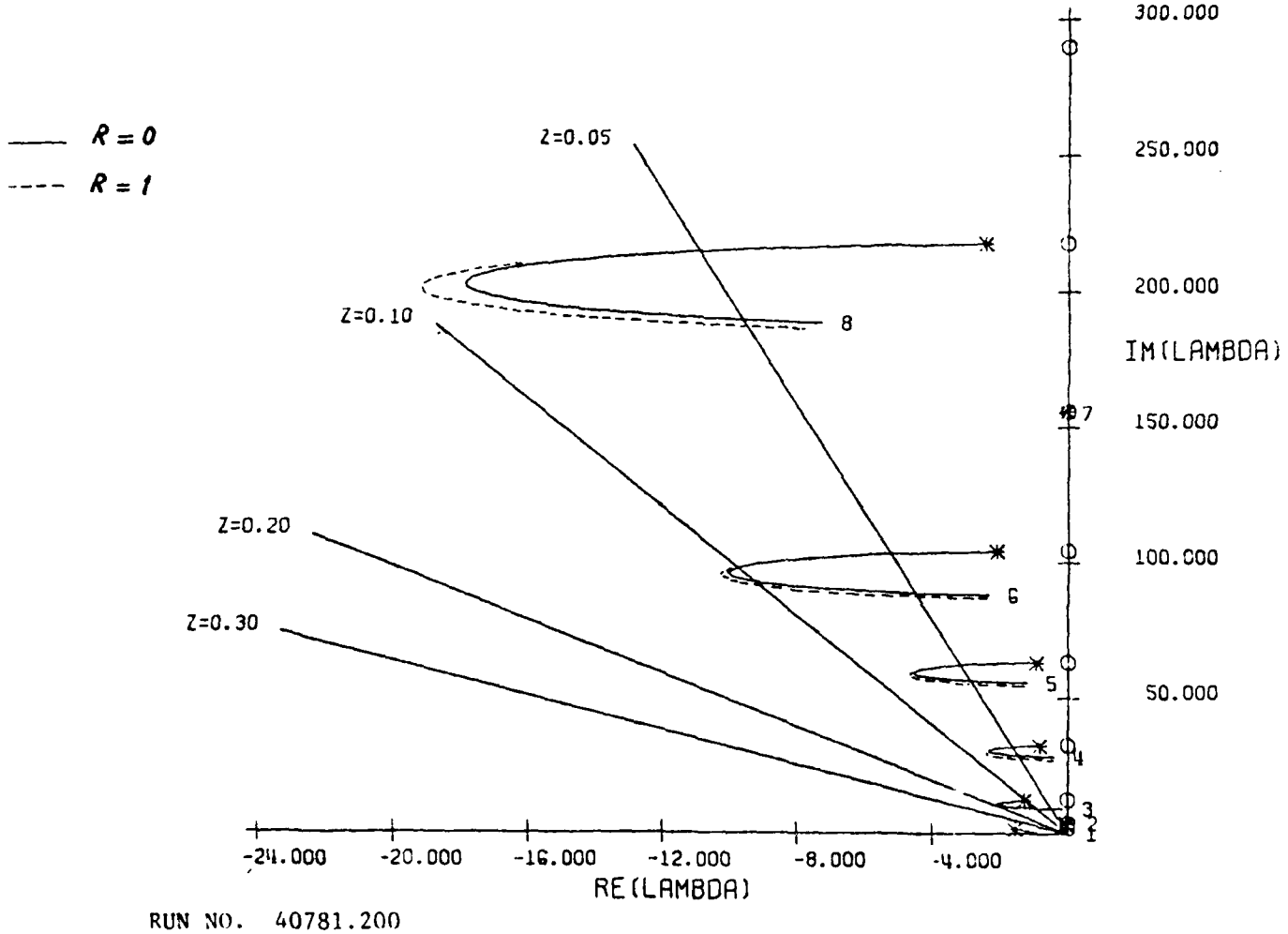
E = 10 x 10^6 lb/in^2 (68.93 GN/m^2), Alluminum

$0.1 \leq c \leq 4.0$ lb.sec./in ($0.0175 \leq c \leq 0.70$ KN.sec/m)

Original system natural frequencies: Ω_1 = 2.065 rad/sec, Ω_2 = 3.804, Ω_3 = 12.504, ℓ_4 = 32.472, Ω_5 = 63.211, Ω_6 = 104.332, Ω_7 = 155.792, Ω_8 = 217.587

Figure 1 Euler Bernoulli beam model for space structure reanalysis

R = 0			R = 1		
MODE	OPTIMUM DAMPER	OPTIMUM ZETA	MODE	OPTIMUM DAMPER	OPTIMUM ZETA
1	.100	.71919	1	.100	.71922
2	.100	.01470	2	.100	.01470
3	.200	.18230	3	.200	.18243
4	.400	.07575	4	.400	.07602
5	.700	.07624	5	.700	.07723
6	.700	.10437	6	.700	.10665
7	1.400	.00190	7	1.500	.00201
8	1.100	.08778	8	1.200	.09508



RUN NO. 40781.200
DAMPER NODE NO. 11
NO. DAMPER VALUES 40
SMALLEST DAMPER(*) .100 LB.SEC.IN^{-1} (0.0175 kN.SEC.m^{-1})
DAMPER INCREMENT .100 LB.SEC.IN^{-1} (0.0175 kN.SEC.m^{-1})
MODES FOR RECEPTANCE 12

Figure 2 Root locus plot for space structure beam model utilizing eigenvalue reanalysis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